MATH PROBLEM OF THE MONTH (April)

Solution:

26. 51/100. Note that each factor $(1 - 1/n^2)$, the difference of two squares, can be written as $(1 - 1/n)(1 + 1/n)$. The problem, then, becomes that of simplifying the following product:

$$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\cdots$$

$$\left(1-\frac{1}{49}\right)\left(1+\frac{1}{49}\right)\left(1-\frac{1}{50}\right)\left(1+\frac{1}{50}\right)$$

$$=\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)\left(\frac{5}{4}\right)\cdots$$

$$\left(\frac{48}{49}\right)\left(\frac{50}{49}\right)\left(\frac{49}{50}\right)\left(\frac{51}{50}\right)$$

$$=\left(\frac{1}{2}\right)\left(\frac{51}{50}\right)$$

$$=\frac{51}{100}$$