MATH PROBLEM OF THE MONTH (April)

Solution

The answer is 1731.
Let that whole number be x. When x is divided by 5, remainder is 1, when divided by 7, remainder is 2, when divided by 9, remainder is 3, and when divided by 11 remainder is 4.

We write this as:
x = 1 mod 5
x = 2 mod 7
x = 3 mod 9
x = 4 mod 11

We can observe that the remainder increases by 1 as the divisor increases by 2. If we multiply x by 2, then the remainders would each double and thereby also increase by 2 as the divisor still increases by 2. This gives us:
$2x$ = 2 mod 5
$2x$ = 4 mod 7
$2x$ = 6 mod 9
$2x$ = 8 mod 11

Now we notice that all the remainders are 3 less than the divisors. So, if we add 3, we will have remainders of zero. Thus, $2x + 3$ is zero in modulo of all four divisors. Finally, since 5, 7, 9, and 11 have no common factors, the smallest value of x possible is found with:
$2x+3=5\times7\times9\times11$, which leads to $x=1731$.